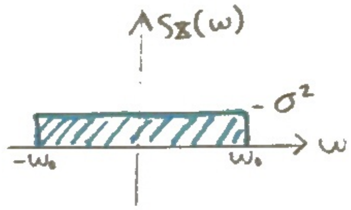
Föreläsning 5/12-13

$$\underbrace{S_X(\omega)}_{\text{PSD}} = \int_{-\infty}^{\infty} e^{-i\omega\tau} \underbrace{R_X(\tau)}_{\text{autocorrelation}} d\tau \qquad R_X(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} S_X(\omega)\, d\omega$$

$$R_X(\tau) = e^{-|\tau|} \Rightarrow S_X(\omega) = \int_{-\infty}^{\infty} e^{-i\omega\tau} e^{-|\tau|} d\tau = \int_0^{\infty} e^{-(i\omega+1)\tau} d\tau + \int_{-\infty}^{0} e^{-(i\omega-1)\tau} d\tau =$$

$$= \frac{1}{i\omega+1} + \frac{1}{1-i\omega} = \frac{2}{1+\omega^2}$$

$$R_X(\tau) = \frac{2}{1+\tau^2} \Rightarrow S_X(\omega) = 2\pi e^{-|\omega|}$$

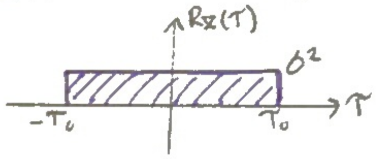


$S_X(\omega)$ is bad limited white noise

$$R_X(\tau) = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} e^{i\omega\tau} \sigma^2 d\omega =$$

$$= \frac{\sigma^2 (e^{i\omega_0\tau} - e^{-i\omega_0\tau})}{2\pi i \tau} = \frac{\sigma^2 \omega_0}{\pi} \operatorname{sinc}(\omega_0 \tau), \quad \operatorname{sinc}(x) = \frac{\sin x}{x}$$

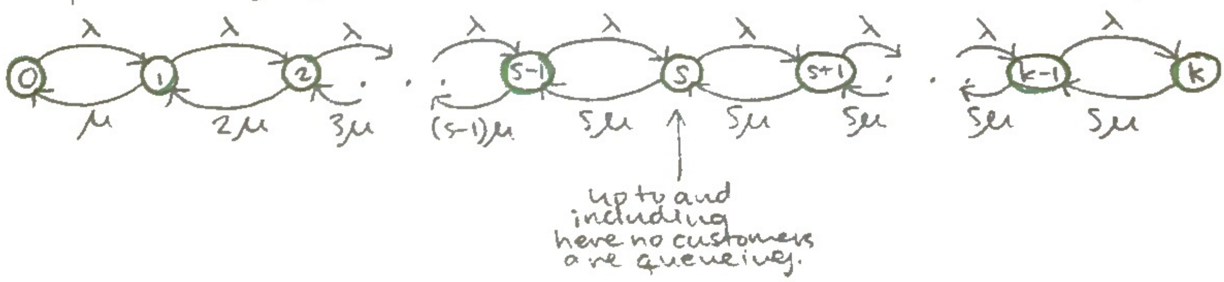
$R_X(\tau) =$



$$\Rightarrow S_X(\omega) = 2\pi \frac{\sigma^2 \tau_0}{\pi} \operatorname{sinc}(\tau_0 \omega)$$

Chapter 9 in Hsu Queues

Customers arrive according to Po-process with intensity λ (mean $\exp(\lambda)$-dist. times between arrivals) to queueing system with s servers which each use an $\exp(\mu)$-dist. time to serve a customer. The total number of people in queueing system (i.e number of people being served + number of people waiting in queue for service.) is limited to k (where $k=\infty$ is possible). Total number $X(t)$ of people in queueing system at time t is birth-and-death process.



People that arrive to full queueing system ($X(t)=k$) just bounce away (do not enter system).

We study these kind of queueing systems when $\mu^{(0)} = \pi$, $P(X(0)=n) = \pi_n$. Then we want to find the following six characteristics:

L - the average number of customers in whole queueing system.
L_q - ——— '' ——— that are queueing.
L_s - ——— '' ——— that are being served.
W - the average amount of time spent in system for typical customer.
W_q - ——— '' ——— queueing for service ——— '' ———
W_s - ——— '' ——— in service ——— '' ———

$$L = L_q + L_s$$

$$W = W_q + W_s$$

$$W_s = 1/\mu$$

$$L = E(X(t)) = \sum_{n=0}^{k} n \pi_n$$

$L = W\lambda$ for $k = \infty$

$L = W\lambda(1 - P_k)$ for $k < \infty$

same is true for L_q, W_q, L_s, W_s. Only two of these three equations give something new as the third follows from the two other.